

# A Queuing Model to Optimize the Performance of Surgical Units

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**Abstract**— In any country healthcare service takes a major place which provides care for persons who are not well. When considering the Healthcare service in Sri Lanka, National Hospital of Sri Lanka plays a major role. The hospital faces the problem of overcrowding the patients and long waiting queues of patients for different healthcare needs specially on surgical purposes. The objective of the study is to introduce a queuing model for optimizing the performance of surgical units in National Hospital of Sri Lanka. In the study, the operation theaters of Accident and Emergency unit were only considered. According to the queuing model M/M/3 the existing system is not utilized well due to arrival rate is higher than the service rate. Hence it is needed to increase the service rate. According to the current system the theaters function in 4 equal work shifts in 24 hours. Some operation theaters do not function on all the shifts. It was identify that the average maximum service rate of one shift is 4.25 patients. By making all theaters functioning in that rate in all shifts the system can make efficient with the service rate of 51 patients per day. This task will be achieved by increasing the capacity of live resources.

**Keywords**— M/M/3 model, Operation Theater, Queuing theory, Surgical units

## I. INTRODUCTION

Overcrowding in healthcare systems such as hospitals is a problem worldwide and affects the ability to provide emergency medical care within a reasonable period of time. Surgical units represent one of the most critical and expensive hospital resources since a high percentage of the hospital admissions is due to surgical interventions. Surgical units can be regarded as a network of queues and different types of servers where patients arrive, some patients wait for the list, get admitted to the hospital, wait for a service, undergo the surgery, get recover and then go home. Long waiting time in any hospital is considered as an indicator of poor service performance and needs improvement. If the waiting time and service time is high patients may leave the queue prematurely and this in turn results in patients dissatisfaction. Hence it is very important to manage the queues in surgical units. In the

other words find the balancing point where patients and the service providers may get satisfied.

Sri Lanka is a developing country and the National Hospital is the heartiest free healthcare service provider in the country. Hence the hospital faces the problem of overcrowding the patients and long waiting queues of patients for different healthcare needs specially on surgical purposes. In the study, the operation theaters of Accident and Emergency unit were only considered. In the current system of the Accident and Emergency unit consists with three operation theaters. Patients of ward 72 and 73 who come for surgical purposes arrive to theater in one queue and distributed among the three.

### 1.1 MOTIVATION

The National Hospital of Sri Lanka consists of eight surgical units. In Sri Lanka Most of the patients keep their trust on the National Hospital for their surgical purposes. Therefore there is a critical congestion and long waiting queues in surgical units. Overcrowding in surgical units and long waiting lists are a problem and affects the ability to provide emergency medical care within a reasonable period of time. For each surgical unit it is a challenge for the management to manage the congestions and long waiting queues to reduce the patients' waiting time for their surgery, and to improve the patient's satisfaction. Hence the study is based on the Accident and Emergency unit of National Hospital of Sri Lanka.

### 1.2 OBJECTIVES

The main objective of the study is to investigate how queuing models can be used to reduce the patients' waiting time in the list of patients and waiting time in the hospitals for their surgeries at Colombo National Hospital. The initial objective of the study is studying the situation gathering rich data. This study always evaluate the surgical unit service system such as patients' arrival rates, the average time that a patient has to wait in the queue for each operation theatre and the service rate. The study will also examine whether the resources of the surgical units are adequate to provide an effective and

efficient service to patients and how the utilization of the resources can be optimized.

## II. PROBLEM DESCRIPTION

All the patients who get admitted to the Accident and Emergency unit in the hospital should give the same priority. But in present situation patients have to wait in queues to fulfill their surgical purposes. The arrows of the following figure illustrate the waiting queues of the surgical units that patients have to wait.

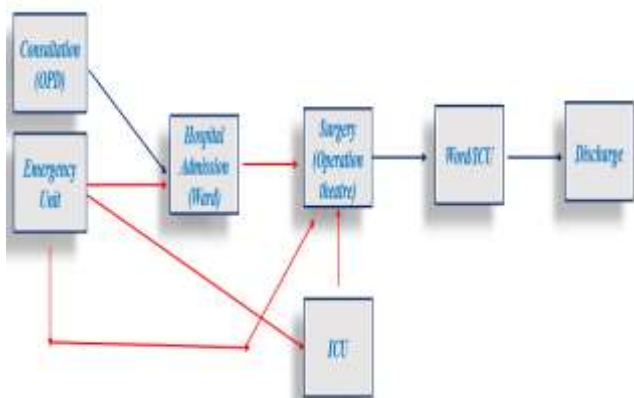


Fig. 1: The queuing network of the system

The study is based on the waiting queues which are indicated in red arrows.

## III. LITERATURE REVIEW

Many of the researchers have applied queuing models to analysis the patient flow and optimize the patient flow in health care service systems in many other countries. Unfortunately in Sri Lanka there are no many research findings regarding the patients flow optimization in healthcare service systems. There is one study has base on National Hospital of Sri Lanka. In 2016 Dilrukshi, Nirmanamali, Lanel and Samarakoon have conducted a study, "A Strategy to Reduce the Waiting Time at the Outpatient Department of the National Hospital in Sri Lanka". This study has analyzed the patients flow in OPD. There is no more any clear evidence that the researchers have worked on analyzing the patients flow in National Hospital of Sri Lanka. Similarly there is no any significant study based on the surgical units of National Hospital. There are critical congestions in these units since many patients registered and get admitted to the hospital due to surgical purposes. Hence it is important to introduce an improved approach for optimizing the performance of the surgical units in National Hospital in Sri Lanka in order to increase the qualitative index of the healthcare service in Sri Lanka. This study will conduct to fulfill the above purposes. This project will identify the existing situation of the surgical units and measures the existing performance of the units by using queuing

models. Then the study will carry out for the necessary implementations.

## IV. METHODOLOGY

There are two surgical wards (Ward 72 and 73) for Accident and Emergency department and three operation theaters. Considering one month daily data records of the wards and the operation theaters and the data of resources of above theaters and wards the analysis has carried.

Initially the existing conditions and the current practices were observed to identify the present situation. Then using the simple Descriptive Statistics techniques arrival rate of the patients' service rates of the operation theatres was calculated. Then the average functioning pattern of operation theaters and the average arrival pattern of the patients were identified.

Queuing theory has its origins in research by Agner Krarup Erlang when he created models to describe the Copenhagen telephone exchange. Queuing theory is the mathematical study of waiting lines, or queues. In queuing theory, a model is constructed so that queue lengths and waiting time can be predicted.

In queuing theory the arrivals can be either constant or random. If arrivals are random, it assumed that it follows poisson's distribution. Service patterns are like arrival patterns in that they may be either constant or random. If service time is constant, it takes the same amount of time to take care of each customer. More often, service times are randomly distributed. In many cases, it can be assumed that random service times are described by the negative exponential probability distribution.

Here the following queuing model has applied to the problem to reduce waiting time of a patient. Since the population is unlimited, the queue is infinite, and the queuing system is multichannel, hence the multichannel with single phase model (M/M/C) was selected for the further analysis.

The following queuing parameters and formulas were used for calculations.

Table.1: Queuing parameters and their formulas

$\lambda$ – Average arrival rate	$a = \frac{\lambda}{\mu}$
$\mu$ – Average service rate at each server	$\rho = \frac{\lambda}{\mu C}$
$L_q$ – The average number of patients waiting in the queue	$P_0 = \left( \sum_{K=0}^{C-1} \left( \frac{a^K}{K!} \right) + \left( \frac{a^C}{C!} * \frac{1}{1-\rho} \right) \right)^{-1}$
$L$ – The average number of patients in the system	$L_q = P_0 * \frac{a^C}{C!} * \frac{\rho}{(1-\rho)^2}$
$\rho$ – The system utilization	$W_q = \frac{L_q}{\lambda}$
$W_q$ – The average time a patient spends in the queue	$W_s = \frac{1}{\mu}$
$W$ – The total time a patient spends in the system	$W = W_s + W_q$
$W_s$ – Service time	$L = W\lambda$
$P_0$ – The probability of zero patients in the system	
$C$ – The number of servers (channels)	

## V. RESULTS

### 5.1 BASIC RESULTS

As the initial step the current situation is observed and the arrival and service rates were calculated in both wards and the operation theatres. The following table illustrates the summary records.

Table.2: Average no. of patients arrivals per day

Ward	Admission	Discharge	Total	Surgery
72	68	63	76	41
73	27	19	45	7

According to the existing system patients arrival rates in hourly time period for the operation theater has the following distribution.

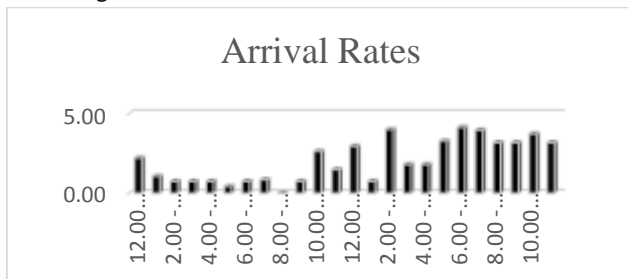


Fig. 2: Arrival rates of the patients (Arrival rates within time)

There are three operation theatres. The arrival and service rates of the operation theaters (OT) were calculated. The following table illustrates the summary records.

Table.3: Arrival rates and Service rates of the Theaters

	Arrival Rate	Service Rate
OT I	14	13
OT II	14	12
OT III	20	17
<b>Theater</b>	<b>48</b>	<b>42</b>

The service rates of the each theater were analyzed. The following figures described the service rate distribution of the each theater in hourly time period.

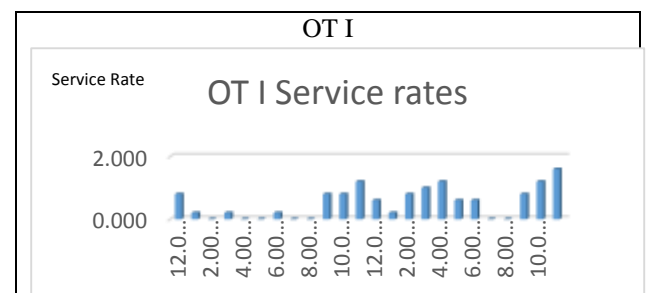


Fig. 3: Service rate pattern of OT I

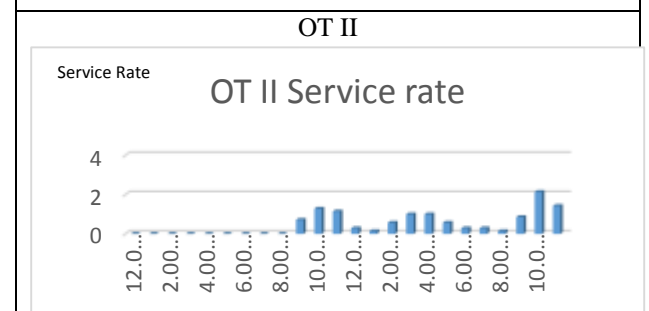


Fig. 4: Service rate pattern of OT II

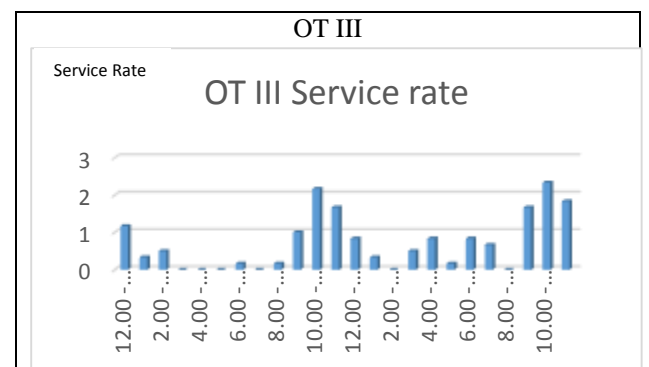
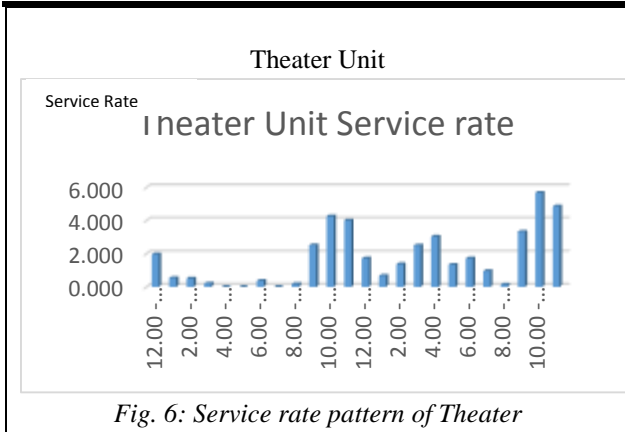


Fig. 5: Service rate pattern of OT III



By using M/M/3 queuing model the system utilization ( $\rho$ ) was calculated.

$\rho = 1.142 > 1.00 \rightarrow$  The system is not utilized well.

Hence the other parameters cannot be calculated.

## 5.2 SUGGESTIONS AN IMPLEMENTATIONS

According to the existing system the theaters are functioning in four shifts. 8.00am - 2.00pm, 2.00pm - 8.00pm, 8.00pm - 2.00am, and 2.00am-8.00am, but OT II does not function on the 2.00am-8.00am shift. The physical capacities of all three theaters are equal. Hence the suggestions will be made with minor affect to the current practice. Due to the physical capacities of all theaters are equal, it is assumed that all theaters can perform in same level in each shift. By considering the current performance the average rate of service of the operation theaters in a shift are illustrated in the following table.

Table.4: Existing Average service rates for 6 hours shift

Operation Theater	Service Rate (patients)
OT I	3.25
OT II	4
OT III	4.25

The highest average rate is 4.25 patients per shift. Then it was assumed that the theaters can perform with the average service rate of 4.25 for a shift. There are three theaters and four shifts for each theater. Then the service rate of the Operation Theater of the day will be 51 patients. The arrival rate is 48 which is less than service rate.

By using M/M/3 queuing model the system utilization ( $\rho$ ) was calculated.

Table.5: Queuing model parameters

Paramerts	Values (per day)
$a$	2.82353
$\rho$	0.94118 < 1 The system is efficient
$P_0$	0.01417
$L_q$	14.45647
$W_q$	0.30118 (7.228 hours)
$W_s$	0.05882 (1.411 hours)
$W$	0.36 (8.64 hours)
$L$	17.28

By making all three shifts to perform in same capacity the system can be made efficient with the existing physical resources. Using this model OT I and OT II function on that shift in the average service rate of 4.25 the system can be made efficient. Then the task is how make OT I and OT II to achieve the average service rate of 4.25 patients during the shift 2.00am – 8.00am. OT I still function on that shift. But OT II is not. Then the live resources should be allocated to the theater on the shift. The existing live resources capacities of the system are given below.

Table.6: Existing live capacities of the operation theater

OT II & OT III				
	No. of Surgeons	Doctors	Nurses	Minor Staff
OT Capacity	6	9	33	30
One shift	1	2	2	2

OT I				
	No. of Surgeons	Doctors	Nurses	Minor Staff
OT Capacity	4	6	28	23
One shift	1	2	2	2

Considering OT I and OT II combination, for a day there are 5 work shifts are functioning in the current. In the proposed system it increased up to 6. The staff roster is made for a one month period. By considering one month period (30 days) there are 150 shifts in a month. The average numbers of shifts for the staff are shown below.

Table.7: No. of working shifts for a month

OT II & OT III				
	Surgeons	Doctors	Nurses	Minor Staff
No. of Shifts	25	16.67	4.55	5

In the proposed system there are 6 work shifts, in other words there are 180 work shifts for a month. Without disturbing the current average number of shifts that an operation theater service provider has to work, the no. of

employees need to perform the proposed system is shown below.

Table.8: Proposed work capacity

	OT II & OT III			
	No. of Surgeons	Doctors	Nurses	Minor Staff
proposed capacity	7.2	10.80	39.60	36
	8	11	40	36

Then OT II and OT III will be able to achieve the proposed service target.

Then attention moved towards the OT I. The arrival rate of the OT I is comparatively low to other two theaters. By making patients distributing among the theatres equal and by increasing the service rate the target can be achieved. Furthermore OT I is not function as a practices in 2.00-3.00 am, 4.00-6.00am, 7.00-9.00 am, 7.00-9.00 pm. By making those time slots to functions the service rate can be increased.

## VI. CONCLUSION

In the study, the operation theaters of Accident and Emergency unit were only considered, because the priorities of the all patients are equal.

It was found out that the queuing model of the existing system is M/M/3 model because the system has 3 theaters. By using M/M/3 queuing model the system utilization ( $\rho$ ) was calculated. The  $\rho$  value is 1.142 which is greater than 1.00. it implies that The system is not utilized well.

The system can be made utilized well by increasing the service rate. It was found that the each theater should perform in the rate of 4.25 for 6 hour time period.

To achieve this service rate the operation theater staff carder should be enhanced. It was identified the staff carder of OT II and OT III same. Due to OT II is not functioning in 2.00am-8.00 am work shift; in order to make it function the work carder should be maximized.

Then by making above mention changes the system can be made efficient.

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